

## Homework 3

*Lecturer: Santosh Vempala**Due Date: 17 Nov 2021*

Notes:

- You can discuss and collaborate, but please write your own solutions, and clearly mention everyone you discussed with.
- Start on a new page for each problem.
- Submit on Canvas via Gradescope

**1. [Boolean Functions]**

1. Let  $n$  be an odd positive integer. The  $MAJ_n$  function  $f : \{-1, 1\}^n \rightarrow \{-1, 1\}$  evaluates to 1 or  $-1$ , whichever has the higher frequency in the input  $x \in \{-1, 1\}^n$ . Find all the discrete Fourier coefficients of  $MAJ_3$  and  $MAJ_5$ .
2. Recall that a function is odd if  $f(x) = -f(-x)$  for all  $x$ , and it is called even if  $f(x) = f(-x)$  for all  $x$ . Show that any Boolean function  $f$  can be written as the sum of an odd function and an even function. Give an example of an odd Boolean function and one of an even Boolean function.
3. Show that for an odd Boolean function, all its even Fourier coefficients, i.e., the coefficient for subsets of even size, must be zero, and for an even Boolean function, all its odd Fourier coefficients must be zero.

**2. [SQ algorithms]**

1. Show how to implement Perceptron and kernel Perceptron as Statistical Query algorithms. [Hint: instead of using a single misclassified example in each iteration, use the average of all misclassified examples; implement the latter as a Statistical Query.]
2. Show how to approximate the top eigenvalue of the covariance matrix of a distribution by a Statistical Query algorithm.