CS7545: Machine Learning Theory **Fall 2021**

Homework 0

Lecturer: Santosh Vempala *Due Date: 30 Aug 2021*

Notes:

- This first homework is to be done entirely on your own, without consulting classmates or anyone else.
- Submit on Canvas via Gradescope

1. Predict the series

What are the next few terms? If possible give a short formula or algorithm for the n'th term. This can be direct/explicit or recurrent.

- $1, 3, 13, 183, 33673, ...$
- $1, 2, 7, 14, 43, 86, 259, 518, ...$
- $1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 7, 7, 8, 9, 10, 10, 11, 11, 12, 12, 12, 13, 13, 13, 14, 14, 14, 14, ...$

2. Place your bet

A race track has the same two horses each day, called H1 and H2. Exactly one of them wins each day. Consider three possible strategies:

- Always bet on H1
- Always bet on H2
- Always bet on whichever horse has more wins so far, picking H1 if both have the same number of wins.

Show that, over a period of T days, the third strategy can be worse than both H1 and H2, and that the number of losses for the third strategy can be nearly twice as many as the losses of either H1 or H2.
(Bonus) Propose a better strategy that would have fewer losses.

3. Represent

A Boolean formula is a function that takes as input Boolean variables and using only the operations AND, OR (on pairs) and NOT (on a single input) produces a single Boolean output.
A decision list is a rule on n input Boolean variables $x_1, x_2, \dots, x_n$ of the following form: if x_1 then FALSE else if $\overline{x_2}$ then TRUE else if x_3 then ...

- Show that any decision list on n variables can be written as a Boolean formula.
- Give an example of a Boolean formula that cannot be written as a decision list on the same variables.

4. Find your kernel

Given some domain $X \subseteq \mathbb{R}^n$, a *kernel* is pairwise similarity function $K(x, y)$ for pairs of points $x, y \in X$, s.t. there exists some mapping $\phi : \mathbb{R}^n \to \mathbb{R}^m$ that represents K as an inner product in the mapped space, namely $K(x, y) = \phi(x) \cdot \phi(y)$. Show that if K_1, K_2 are kernels, then so are $K_1 + K_2$ and K_1K_2.