Undecidability.

Tuesday, September 3, 2019 5:52 PM

This, and the two upcoming lectures, are about <u>non-existence</u>.

Basic decision problem:

$\boxed{\text{Does } x \in L\text{ ?}}$ $\quad x \in \Sigma^*$

e.g. if L = strings with an even # 1's

there is a DFA to <u>decide L</u>.

if $L = \{\langle G=(V,E)\rangle : G \text{ is Hamiltonian}\}$

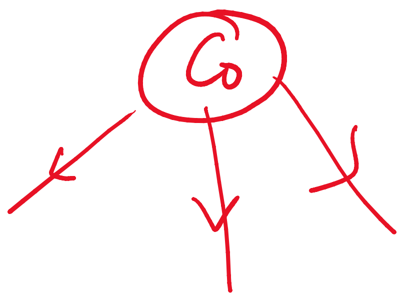
there is a TM to decide L.

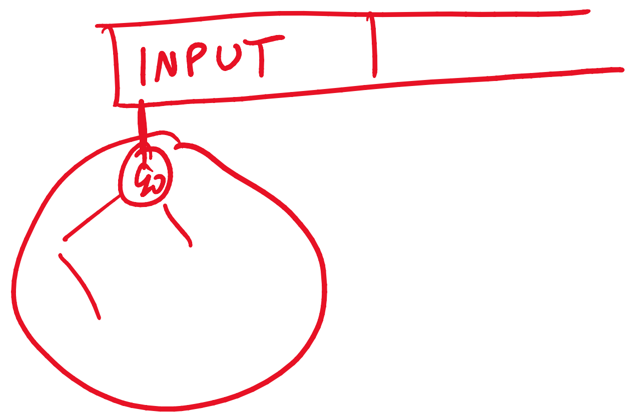
<u>Detour</u>

<u>Thm</u>. <u>NTM $\Rightarrow$ Deterministic TM</u>

<u>Pf.</u> Consider the computation tree of

17. Consider the computation tree of the NTM. Each configuration has ⟨tape contents, head position, state⟩ and has possible transitions





Recall an NTM accepts if

$\exists$ path to some accepting configuration.

Q. $\exists$ path from C_0 to accepting C in computation tree ?

How to solve this? DFS? BFS?

... path could go on

DFS: bad idea, as path could go on forever.

BFS: visits all configurations in increasing order of distance.
So it will reach accepting C at finite distance if one exists!

$$\boxed{x \in L?}$$

Q.
Is every language decidable by some TM?

survey says: ....
What language might be hard to decide?
TSP? No. 2-player game? No.

A set L is countable / enumerable if
$\exists f : L \to \mathbb{N}$ so that every element

$\exists f : L \to \mathbb{N}$ so that

of L has a natural number assigned to it.

Then we can read off L in that order,

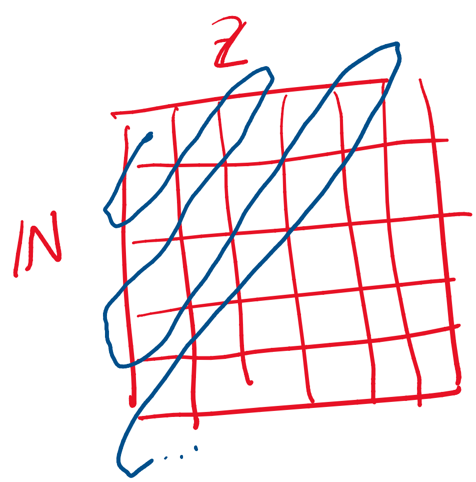
i.e., enumerate it.

E.g. is $\mathbb{Z}$ countable?

$$f(z) = \begin{cases} 2|z| & \text{if } z \leq 0 \\ 2|z| - 1 & \text{if } z > 0 \end{cases}$$

What about $\mathbb{Q}$, set of rational numbers?

YES!



A language is a set of strings.

Σ^* is countable. Why?

$L \subseteq \Sigma^*$ is countable. Why? use

So any $L \subseteq \Sigma^*$ is contable. Why? use same map!

What is a TM description? Fix Σ.

$\langle Q, \delta, q_s, F \rangle$

this is a string!

$\therefore$ set of all TM programs is contable.

What about set of all languages?

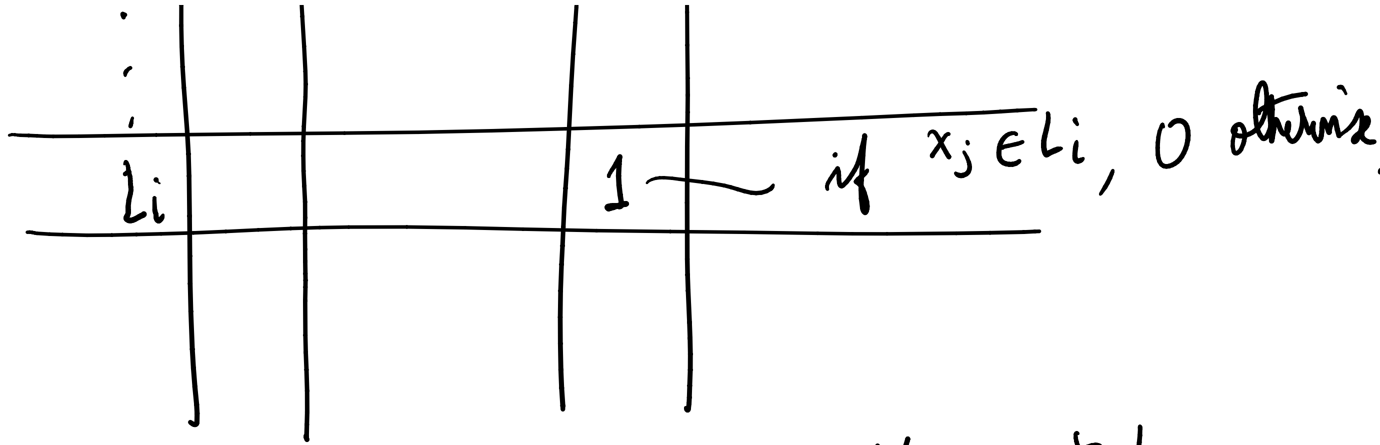
<u>Thm</u>. Set of all Languages is <u>NOT</u> contable.

<u>Pf</u>. Suppose it is. $\exists$ ording $L_1, L_2, \ldots$

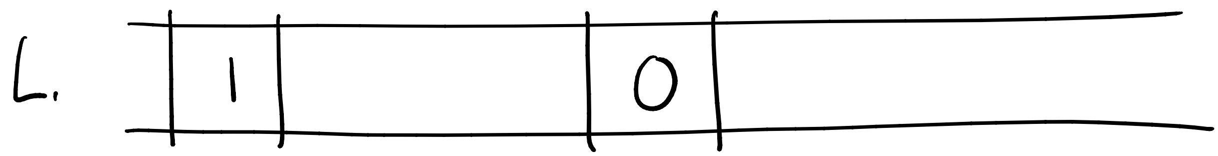
Also Σ^* is countable, so $\exists$ ordering of strings

$x_1, x_2, \ldots$

	x_1	x_2	.	x_j	.
L_1	0				
L_2					
$\vdots$					



Define a new L: $x_j \in L$ iff $x_j \notin L_j$



Since Languages are countable L must have some index k, $L = L_k$.

Q. Does $x_k \in L_k$?

!!! $x_k \in L_k \iff x_k \notin L_k$.

$\Rightarrow$ Contradiction.

Our assumption that languages are countable must be false! $\blacksquare$

Col. $\exists$ languages that cannot be decided

Col. $\exists$ languages that cannot be decided by TM's.

Q. such as ?