

Reductions

Tuesday, November 5, 2019 8:33 PM

CLIQUE

IND. SET

HAM

VERTEX COVER

ILP

SAT

⋮

$P \subseteq NP \wedge \text{co-NP}$

$L \in P \Rightarrow \forall x$ if $x \in L$ there is a
poly size certificate

if $x \notin L$ _____ .

Reduction

$A \rightarrow B$

$x \in A? \text{ --- } y \in B?$

... efficiently given access

A can be solved efficiently given access to a solver for B.

Ex. 1. IND. SET \rightarrow CLIQUE
 $G = (V, E) \exists$ IND. SET of size $\geq k$?
 \Updownarrow
 $\bar{G} = (V, \bar{E}) \exists$ clique of size $\geq k$?

Ex. 2. CLIQUE \rightarrow IND. SET.
Time for reduction? polynomial!

Ex. 3
V.C. \rightarrow I.S.
I.S. \rightarrow V.C.
 $G = (V, E) \exists$ V.C. size $\leq k$?
 \Updownarrow
 \exists ind. set of size $\geq n-k$?

~~(V.C. \rightarrow I.S.)~~

~~(I.C.)~~

Ex. 4 CNF SAT \rightarrow I.L.P.

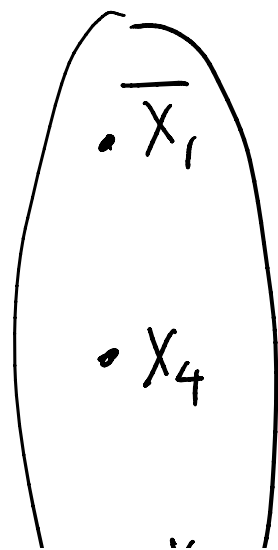
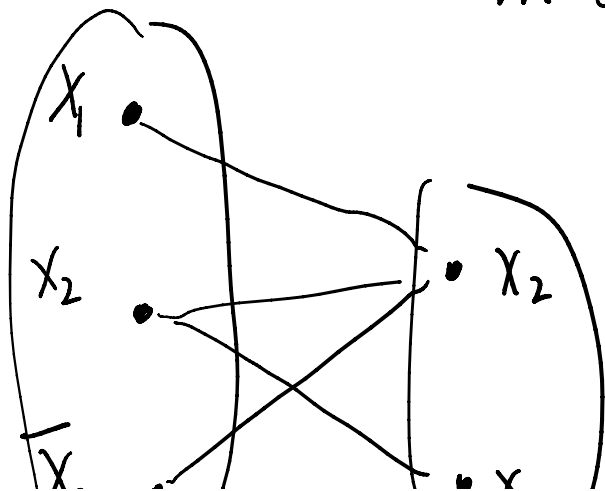
$$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_2 \vee \bar{x}_1 \vee x_4 \vee x_5) \wedge \dots$$

$$\begin{array}{l} 1 = T \\ 0 = F. \end{array} \left\{ \begin{array}{l} x_1 + x_2 + (1-x_3) \geq 1 \\ x_2 + (1-x_1) + x_4 + x_5 \geq 1 \\ \vdots \\ 0 \leq x_i \leq 1. \end{array} \right.$$

Ex. 5 CNF SAT \rightarrow CLIQUE

$$(x_1 \vee x_2 \vee \bar{x}_3) \wedge (x_2 \vee x_3) \dots \wedge (\bar{x}_1 \vee x_4 \vee x_5)$$

m clauses.





$(x_i, x_j) \in E$ unless x_i, \bar{x}_i
 if $x_j \neq \bar{x}_i$ and not corresponding
 to same clause.

Lemma. $F \in \text{SAT} \Leftrightarrow G$ has a clique
 of size m .

$F \in \text{SAT}$. pick one true
 literal from each clause \rightarrow clique.

G has an m clique. must be one from each
 clause.

Set all literals in clique to True.
 then x_i, \bar{x}_i are not both in clique.