Context-free Grammars

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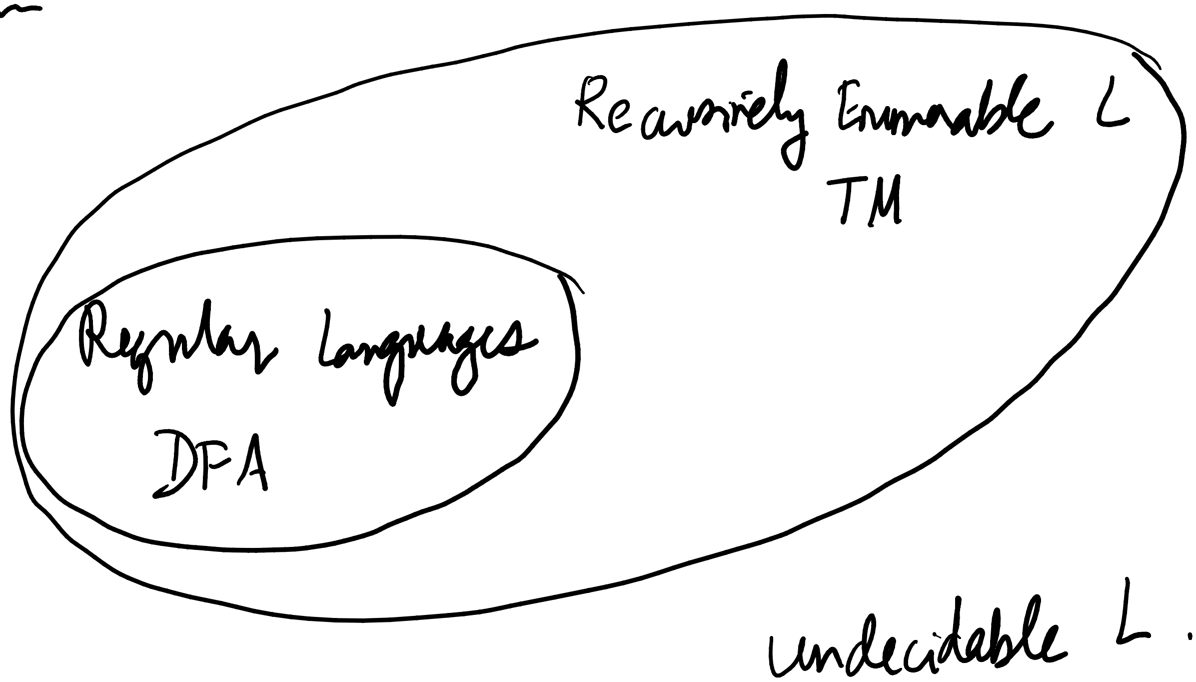


$L = \{0^n 1^n\}$

not regular!

Suppose $\exists$ DFA with p states

By pumping lemma, taking $n = p$

$$0^n 1^n = xyz$$

$$|y| > 0$$

$$|xy| \le p \quad \Rightarrow y = 0^i$$

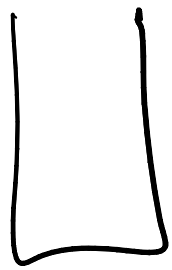
hence $xz = 0^{n-i} 1^n \in L$ X.

Let's give the DFA some power.

Tape? $\longrightarrow$ TM

not that much.

<u>Stack</u>.



<u>push</u> : current symbol on top of stack

<u>pop</u> : remove top symbol of stack.

Q.

q_0

F.

$\delta(q, a, b) \rightarrow q, c$.

while 0 push
 while 1 :
  if stack not empty, pop
  else reject
 if endofinput and stack <u>empty</u>, accept.

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special symbol $ so push to stack first.
when encountered again, stack is empty.

' ' when encountered again, stack is empty.

Ex2. Balanced Parenthesis

( )   (( ))   (( ) ( ))

equal number of (and)

and # (always ≥ #).

DFA ? NO! "(" can lead by a lot.

pumping.

PDA ?

While input,

if (push

if) pop, if empty, reject.

if not empty reject

else accept.

Languages accepted by PDAs?

all?!

What about $\{0^n 1^n 2^n\}$?

- pumping lemma?

What is their power?

More general than regular languages.

$S \to 0S1$

$S \to \varepsilon$

"grammar"

S

0S1

00S11

000S111

000111

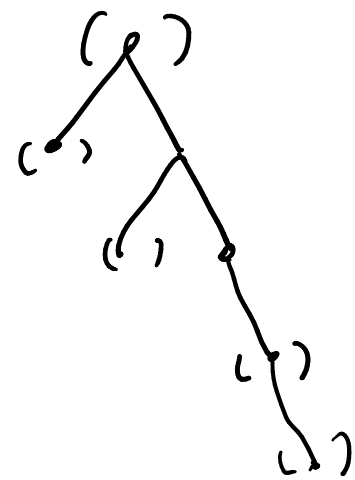
What does this generate? $\{0^n 1^n\}$

What about balanced parenthesis?

$S \rightarrow (S)$

$S \rightarrow SS$

$S \rightarrow \phi$



(( ) ( ) (( )))

CONTEXT FREE GRAMMAR

V: variables (CAPS)

Σ: terminals (small)

R: production rules.

starting variable $S \in V$.

$V \rightarrow (V \cup \Sigma)^*$

$S \rightarrow aBCd$

$B \rightarrow b$

$C \rightarrow Sa$

$S \rightarrow e$.

CHOMSKY NORMAL FORM

$V, \Sigma, R, S \in V$

but each rule is either $A \to BC$

or $A \to a$

or $S \to \varepsilon$.

Equally general: $\forall$ CFG $\exists$ CNF and vice versa.

Do PDAs recognize exactly CFGs?