Space & Time Hierarchies

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- PG

- Asymptotic notation.

$O(f(n))$: "grows no faster than"

$g(n) = O(f(n))$: $\exists\, n_0, c > 0$

$\forall n \geq n_0, \quad g(n) \leq c \cdot f(n)$.

$o(f(n))$: "grows slower than"

$g(n) = o(f(n))$: $\forall c > 0 \;\; \exists n_0$

$\forall n \geq n_0 \quad g(n) \leq c \cdot f(n)$.

- $s : \mathbb{N} \to \mathbb{N}$ is <u>space-constructible</u>

$\exists$ TM that

- $s : \mathbb{N} \to \mathbb{N}$ is ______

if $s(n) \geq \log_2 n$ and $\exists$ TM that on input 1^n outputs $s(n)$ in binary using $O(s(n))$ space

- $t : \mathbb{N} \to \mathbb{N}$ is time-constructible

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using $O(t(n))$ time.

- Space hierarchy theorem.

- Proof.

Time hierarchy theorem?

- break.

Thm. For any time constructible function $t(n)$, Thm. For any time constructible

$\exists L$ s.t. $L \in TIME(O(t(n)))$ but

$$L \notin TIME\left(o\left(\frac{t(n)}{\log t(n)}\right)\right).$$

Pf.. KEY observation: to count up to T needs time $T \log T$. Define a TM D:

- Input $\langle M, 1^n \rangle$
- Start "timer" at $\dfrac{t(n)}{\log t(n)}$.

  decrement every step.

- if "timer" goes to 0, REJECT
- if M is not a valid TM, REJECT

Run M on $\langle M, 1^n \rangle$.

- if M accepts, REJECT

  if M Rejects, ACCEPT.

if M Rejects, ACCEPT.

CLAIM 1: Simulation takes time $O(t(n))$.

L = language accepted by D.

- how to decrement?
- how to follow transitions of M?

Multitrack tape
- timer + M
- working memory

shift along with head!

CLAIM 2: no TM can decide L in $o\left(\frac{t(n)}{\log(t(n))}\right)$ time.

Pf. Suppose $\exists$ TM M_L.

Run D on M_L. takes $o(t(n))$ time.

But D accepts x iff M_L rejects x.

So $L \neq L(M_L)$.