Learning a DFA

Wednesday, October 2, 2019 2:57 PM

- game

ATLANTA → T

SYNEDOCHE → Y

TARHEEL → T

PEACH → P

CHRIS → S

MEMBERSHIP $x \in L$? — YES / NO

is M the DFA ? — YES / NO, counterexample.

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observation Table $\qquad \Sigma = \{0, 1\}$

- rows labeled by strings, candidate states — prefix closed

$$T(s \cdot e) = \begin{cases} 1 & s \in L \\ 0 & \text{o.w.} \end{cases}$$

- columns query strings — suffix closed.

		e		
S		0	1	0
	s	1	0	$\boxed{0}$
		0	0	0
S.A		1	0	0

(1) closed $\quad row(s) \in S \Rightarrow \forall a \in \Sigma \quad row(s \cdot a) \in S \quad (\Leftrightarrow \forall t \in S \cdot \Sigma, \exists s \in S, row(t) = row(s))$

(2) consistent $\quad \forall s_1, s_2 \quad row(s_1) = row(s_2) \Rightarrow \forall a \in \Sigma \quad row(s_1 \cdot a) = row(s_2 \cdot a)$

	ε
S	
S.A	

Algorithm $\quad$ start with $S = E = \{\varepsilon\} \qquad T(\varepsilon)$

$S \cdot \Sigma = \{0, 1\} \qquad T(0), T(1).$

While not closed & consistent:

- If not consistent, i.e. $\exists a \in \Sigma$ s.t. $row(s_1) = row(s_2)$ but $row(s_1 \cdot a) \neq row(s_2 \cdot a)$
  i.e. $\exists e \in E$ s.t. $T(s_1 \cdot a \cdot e) \neq T(s_2 \cdot a \cdot e)$
  add $a \cdot e$ to E and all suffixes. Complete Table
- If not closed, i.e. $\exists t \in S \cdot \Sigma$ s.t. $row(t) \neq row(s)$ $\forall s \in S$.
  add t to S. Complete Table

If closed & consistent. propose a DFA.

$Q = \{row(s) : s \in S\}$

$q_0 = row(\varepsilon)$

$F = \{row(s) : T(s) = 1\}$

$\delta(row(s), a) = row(s \cdot a)$

If counterexample X

add x and all prefixes of x to S

$\delta(row(s), a) = row(s \cdot a)$

Repeat
If conterexample X
add X and all prefixes of X to S.
Complete
else done

Lemma 1. DFA $M(T)$ is consistent with table T and is the smallest such DFA.

Pf. (1) $\delta(q_0, s) = row(s)$

(2) $\delta(q_0, s \cdot e) \in F$ iff $T(s \cdot e) = 1$

induction on $|s|$.

base: $\delta(q_0, \varepsilon) = row(\varepsilon)$

assume (1) for $|s| = k$.

$s = s_1 \cdot a$

$a \in \Sigma$

$$\begin{aligned} \delta(q_0, s) &= \delta(q_0, s_1 \cdot a) \\ &= \delta(\delta(q_0, s_1), a) \\ &= \delta(row(s_1), a) \\ &= row(s_1 \cdot a) = row(s) \end{aligned}$$

Any DFA consistent with T must have distinct states corresponding to distinct rows of S.

Lemma 2. At most $n-1$ conjectures + not closed or not consistent

Time complexity

Target n states, alphabet size k

longest conterexample m

$poly(n, m, k)$ mnk

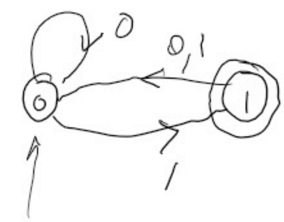
$\Sigma = \{0,1\}$

	ε
S: ε	0
0	0
1	1

closed ?

		ε
S	ε	0
	1	1
	0	0
$S \cdot \Sigma$	10	0
	11	0

closed ? consistent ?



$(0^* 1 \{0,1\})^* 0^* 1$

Counterexample $01 \notin L$

	ε	1	0
ε	0	1	0
1	1	0	0
0	0	0	0
01	0	0	1
10	0	0	1
11	0	1	0
00	0	1	0
010	1	0	0
011	0	0	0

closed ✓

consistent ✓