CFG $\leftrightarrow$ PDA

Tuesday, October 15, 2019 6:54 PM

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CFG

V: variables, $S \in V$

Σ: terminals

R: production rules $\subseteq V \to (V \cup \Sigma)^*$.

PDA

Q: states $q_0 \in Q$, $F \subseteq Q$.

Σ: input alphabet

Γ: stack alphabet

δ: transition relation

$$Q \times \Sigma \times \Gamma \to 2^{Q \times \Gamma}$$

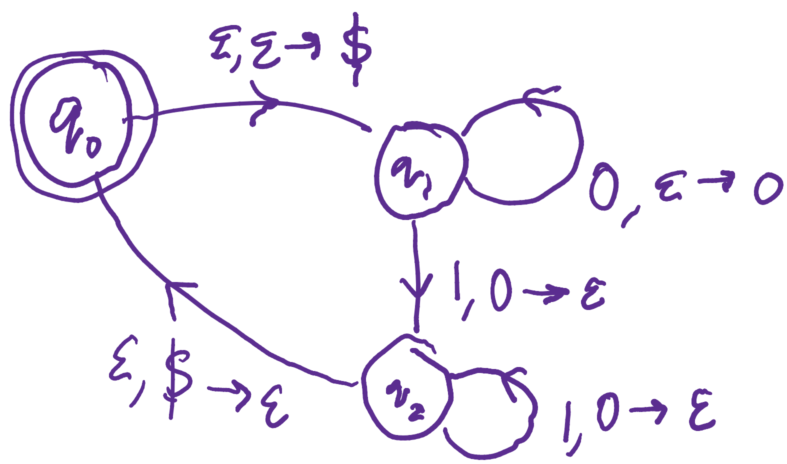
$$\delta(q, \underset{\text{input}}{a}, \underset{\text{top of stack}}{b}) = \{ (q_1, b_1), (q_2, b_2) \ \ldots \}$$

CFG for $(0^n 1^n)^*$

CFG for (0 1)

$$S \to S_1 S \mid \varepsilon$$
$$S_1 \to \varepsilon$$
$$S_1 \to 0 S_1 1$$

PDA:



Q. Is there always a PDA for a CFG and vice versa?

Given CFG, how to construct a PDA?

E.g.

$$S \to SS$$
$$S \to (S)$$
$$S \to \varepsilon$$

input: (()(()))

S

(S)

(SS)

What to keep in stack?
everything in current string to the right of (and including) first variable.

(SS)

((S)S)

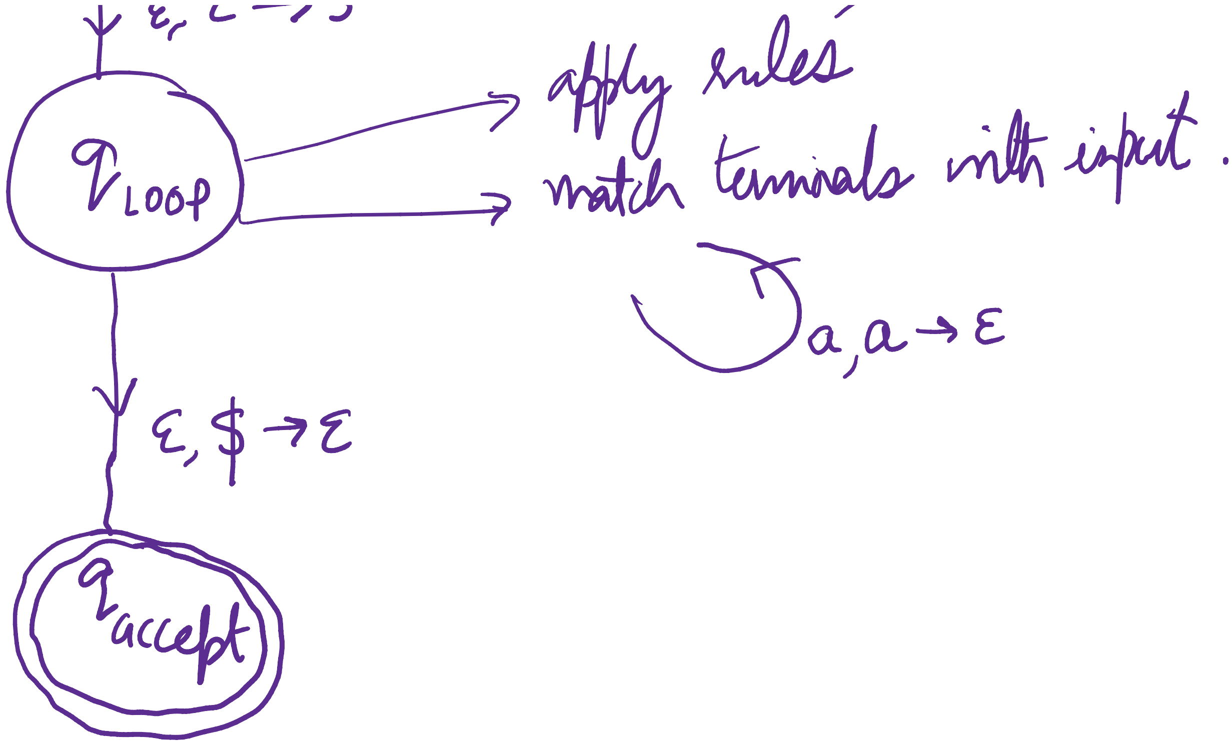
(()S)

(()(S))

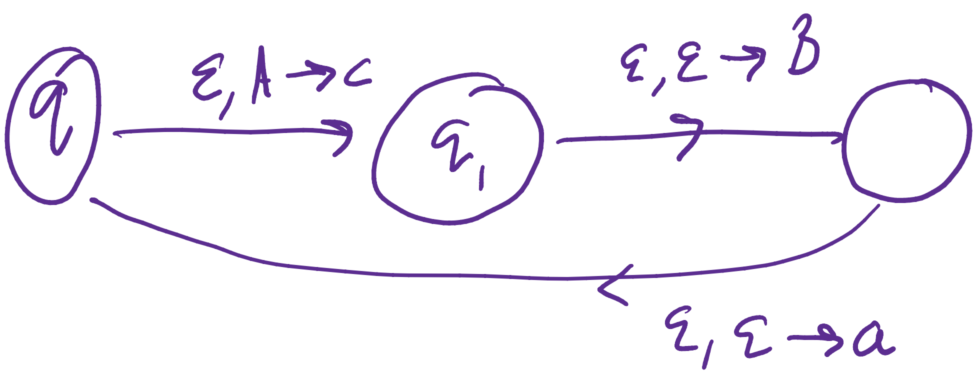
(()((S)))

(()(()))

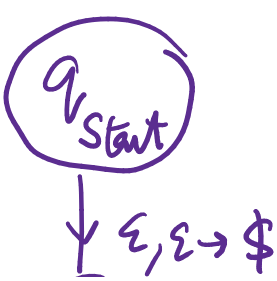
- If top of stack is terminal, match with input.
- If ________ variable, apply rule

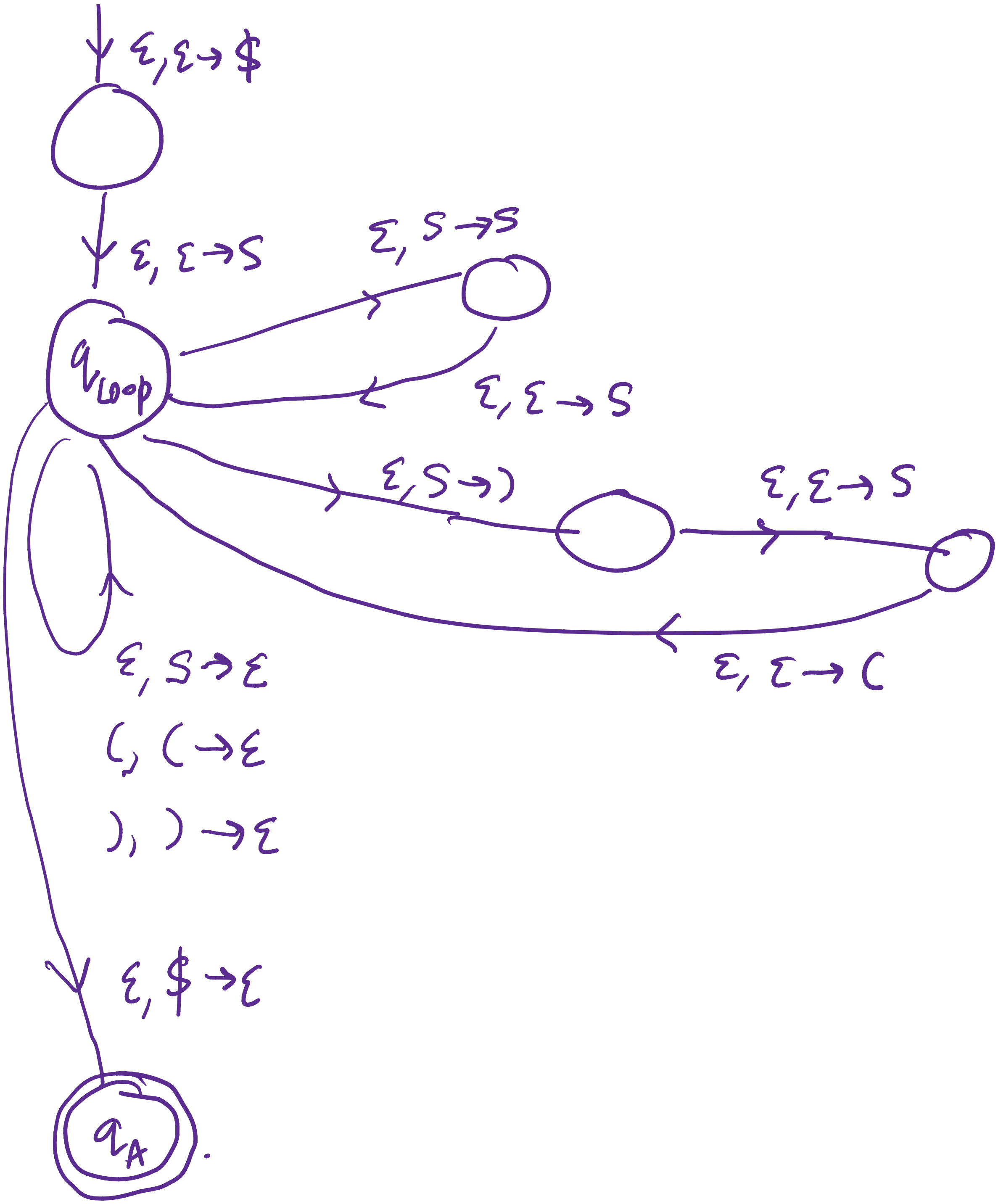


$A \to aBc$



So, for "balanced parenthesis",





ε, ε → $
ε, ε → S
ε, S → S
q_Loop
ε, ε → S
ε, S →)
ε, ε → S
ε, ε → (
ε, S → ε
(, (→ ε
),) → ε
ε, $ → ε
q_A