CS4510: HW8

Due: Nov 18 before noon on Gradescope (there is a link on Canvas)
Separate page for each problem
You should write the solutions on your own,
and include the names of all students you talk to.

1. Dominating Set. [2 points]
Prove that the *Dominating Set* problem is NP-complete.
A subset of vertices S of of an undirected graph G is called a dominating set if every other vertex in G is adjacent to some vertex in the subset S.
Dominating Set $= \{\langle G, k\rangle : G$ has a dominating set with at most k vertices$\}$

2. Cuts. [2 points]
A cut is a partition of vertices of a graph into two disjoint subsets, and the size of a cut is defined as the number of edges crossing the cut.
Given a graph $G = (V, E)$ and an integer k, decide whether there exists a cut of size at most k. Show that the language for this decision problem is in NP and co-NP. Explain your answer.

3. Bow Ties are Cool. [2 points]
A bowtie is a graph on an even number of vertices, say $2k$, in which there are two disjoint cliques of size k with exactly one edge between the 2 cliques.

BOWTIE Problem
Input: An undirected graph $G = (V, E)$ and an integer goal k.
Output: YES if there exists a bowtie of size $2k$ as an induced subgraph of G , i.e., there are disjoint subsets S and T of V which are both cliques of size k and have exactly one edge between them,
or NO, if no bowtie of $2k$ vertices exists in G.
Prove that BOWTIE is NP-complete.