CS4510: HW7

Due: Nov 6 before 3pm on Gradescope (there is a link on Canvas)
Separate page for each problem
You should write the solutions on your own,
and include the names of all students you talk to.

1. Undecidability. [2 points]
Define a useless state of a Turing Machine to be one that is never entered for any input. Show that L = $\{(\langle M \rangle, q) : M$ is a Turing Machine description and q is a useless state of $M\}$ is undecidable.
Hint: Suppose you are given a random Turing Machine T and an input x. Make a new Turing Machine T' out of T, and show that deciding some useless state of T' allows you to decide whether T accepts x.

2. Space and Time hierarchy. [2 points]
Show that $NSPACE(\log n)$ is a *strict* subset of $DTIME(n^{\log^2 n})$.

3. Graph search. [2 points]
Let G be a graph, and let $\alpha \in \mathbb{R}^+$. We call G an α-expander if every $S \subseteq V$ with $|S| \leq \frac{|V|}{2}$ has at least $\alpha * |S|$ vertices outside of S.

a. Show that, for any $u, v \in V$, the distance from u to v is $O(\log_{1+\alpha} |V|)$.
b. The space complexity for determining if there is a path between u and v of length at most k for any two vertices u and v in an α-expander graph G is $O(\log |V| * \log(\log(|V|)))$. Explain.
Hint: Recall the proof of Savitch's Theorem.